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Clusters in Pulsar Distribution

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SUMMARY

According to a pulsar model investigated at Stanford, the pulse-width W should scale with period P as $P^{2/3}$. Moreover, the state of evolution of a pulsar may be characterized by the ratio of the period to the "cut-off period" which scales with age τ as $\tau^{2/3}$. These considerations have suggested that we study the distribution of pulsars in a plane with $\log (P\tau^{2/3})$ as abscissa and $\log (WP^{-2/3})$ as ordinate. This distribution gives the appearance of clustering and we have carried out a statistical test which yields weak evidence for the existence of at least two clusters at an 85% confidence level. However, there is comparatively strong evidence for separation in this plane of pulsars with different pulse shapes.

INTRODUCTION

Considerable theoretical work has been done relating the periods, ages and pulse widths of pulsars (Gunn and Ostriker, 1970; Lerche, 1970; Sturrock, 1971; Roberts and Sturrock, 1972, 1973). An important problem in the study of pulsars is the question of whether pulsars form a single group or whether pulsars come in two or more different groups. Such groups might be related to several factors such as the initial creation of the neutron star, or the orientation of the magnetic field axis with the spin axis ($\vec{\omega} \cdot \vec{M} > 0$ or $\vec{\omega} \cdot \vec{M} < 0$). For example, the pulsar model of Ruderman and Sutherland (1975) has the extreme property that all pulsars have $\vec{\omega} \cdot \vec{M} < 0$; a comparable object with $\vec{\omega} \cdot \vec{M} > 0$ is termed an "antipulsar" which does not radiate.

Sometime ago, T. Gold suggested to J.S. Turk of this University that the period distribution of pulsars may be bimodal. Turk attempted to check this proposal. Since there were no established statistical tests for this problem, tests were devised but these did not give conclusive results.

More recently, Backer (1975) and Roberts (1975) have studied the pulse-width distribution and concluded that no single theoretical distribution adequately fits the data. Since the pulse-width may well be related to the angle between the rotation axis and the magnetic moment, these studies suggest that additional information concerning possible classification of pulsars may be gained by inspecting a two-dimensional distribution involving pulse-width (W) as well as period (P). Since the expected relationship between pulse-width and period is model-dependent, we have pursued an analysis based on the pulsar model currently

being investigated at Stanford (Roberts and Sturrock, 1973). However, such a model-dependent analysis makes it possible to consider not only the parameters W and P but also the "age" τ defined by

$$\tau = P/\dot{P} .$$ (1.1)

In the Stanford model, the pulse-width and period are related (Roberts and Sturrock, 1972) by

$$W \, P^{-2/3} = G \, M^{-1/2} \, R^{1/2}$$ (1.2)

where M is the mass of the neutron star, R is the radius, and G is a geometrical factor determined by the relative orientation of the rotation axis, magnetic axis and line-of-sight.

The condition for radiation is discussed by Sturrock (1971) and in more detail by Sturrock, Baker and Turk (1975). For most pulsars, the condition is expressible as

$$P \, \tau^{2/3} \le 10^{-14.0} \, I^{2/3} \, R^{-1} ,$$ (1.3)

where I is the moment of inertia. (All quantities are expressed in c.g.s. units.)

These considerations have suggested that we study the distribution of pulsars in the plane with coordinates $\log P\tau^{2/3}$ and $\log WP^{-2/3}$ (Figure 1). We have attempted to determine the existence or non-existence of clusters in this distribution. Unfortunately, the necessary information is available for only 72 pulsars, and many common statistical tests seemed unsuitable for discussion of such a small sample.

STATISTICAL PROCEDURE

To determine if the distribution is bimodal we have used an algorithm developed by Sclove (1971). In this algorithm we initially define two bivariate normal distributions by specifying their means and setting the covariance matrices equal to the identity matrix. The pulsars are then divided into two clusters by associating each point with the bivariate normal distribution which maximizes its probability. The group to which each pulsar belongs is defined by

$$
g_i = \begin{cases} 1 & \text{if } P_1(x_i,y_i) > P_2(x_i,y_i) \\ \\ 2 & \text{if } P_2(x_i,y_i) \geqslant P_1(x_i,y_i) \end{cases} \qquad (2.1)
$$

where P_1 and P_2 are the bivariate normal density functions associated with each cluster and $x_i = \log P_T^{2/3}$ and $y_i = \log WP^{2/3}$. Once the points have been assigned to clusters, the mean and covariance matrix of each cluster are estimated by the usual methods. The pulsars are again divided into clusters and the process continues until no point changes cluster. To determine the effect of choosing different starting clusters, we tried several different pairs of initial means. The mean and covariance matrix of the total pulsar population were estimated and the initial cluster means chosen in the following ways: a) a distance $\pm \sigma_1$ from the mean of the total population along the direction of maximum variance; b) a distance $\pm 0.1 \times \sigma_1$ along the direction of maximum variance; c) a distance $\pm \sigma_2$ along the direction of minimum variance; and d) $\pm (\sigma_1 \hat{e}_1 + \sigma_2 \hat{e}_2)$ where σ_1^2, σ_2^2 are the eigenvalues of the covariance matrix and $\hat{e}_1$ and $\hat{e}_2$ are the corresponding

eigenvectors. Choices (a) and (b) resulted in the same clusters; choice (d) resulted in clusters quite similar to (a) and (b). Only choice (c) was significantly different from the others. To determine which of the results (a,b,c, or d) to use, and to determine the significance of the results, we used a likelihood ratio test. The likelihood ratio was defined by

$$\mathcal{L} = L_c/L_o \quad , \tag{2.2}$$

where

$$L_c = \prod_{i=1}^{N} P_c \, (x_i, y_i) \tag{2.3}$$

and

$$L_o = \prod_{i=1}^{N} P_o \, (x_i, y_i) \, . \tag{2.4}$$

In these formulas

$$P_c = \delta_{1g_i} \, P_1 \, (x_i, y_i) + \delta_{2g_i} \, P_2 \, (x_i, y_i) \quad , \tag{2.5}$$

$\delta_{\alpha g_i}$ is the Kronecker delta and g_i is defined by (2.1), and P_o is the estimated bivariate normal probability density of the total population.

The choice of means which produced the largest likelihood ratio was (d) with $\log \mathcal{L} = 114.6$. The clusters and their means are shown in Figure 1. Because of the small number of data points, it was still not possible to determine the significance of this value. The significance was determined by the following procedure.

Five hundred sets of random data were generated. Each set contained 72 points, the same number of points as were in the pulsar distribution, with points chosen from a bivariate normal population distributed with

the same mean and covariance as was estimated for the total pulsar
distribution. Each set of 72 points was then analyzed by the cluster
algorithm in exactly the way the pulsar distribution was analyzed. The
500 likelihood ratios thus produced were then plotted on a histogram
(Figure 2) and compared with the likelihood ratio of the pulsar
distribution. We see that there were 73 sets out of the 500 that had
a likelihood ratio comparable to or greater than 114, the ratio of
the pulsar data. This indicates a confidence level of only 85% for
the existence of clusters in this distribution.

In determining the significance of the clusters we have also used
the pulse-shape types defined by Taylor, Manchester and Huguenin (Huguenin,
Manchester and Taylor, 1971; Taylor and Huguenin, 1971; Taylor and
Manchester, 1975). In Figure 1 note that eight of the ten S- (simple)
type pulsars are in one cluster and all eleven of the C- (complex) ˙
and D- (drifting subpulse) type pulsars are in the other cluster. Since
the clusters have approximately equal populations, the a priori
probability of such a segregation is given by

$$P = 2 \times {}^{10}C_2 \; (\tfrac{1}{2})^{21} \approx 10^{-4.4} \; .$$
(2.6)

Thus we can say with reasonable confidence that the segregation of the
S-type pulsars from the C- and D-type pulsars is statistically significant.
As a check on the physical significance of this fact we have performed
the same analysis using the pulse shapes defined by Roberts (1975) and
Backer (1975). In both of these cases we find no statistically
significant segregation.

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This appears to leave us with two alternative conclusions:

a) The SCD classification scheme of Taylor, Manchester and Huguenin has a physical basis different from the classification schemes discussed by Backer and Roberts, and our clusters are related to the former scheme but not to the latter schemes.

b) The classification scheme of Taylor, Manchester and Huguenin incorporates certain selection factors in the choice of pulsars, and it is this selection scheme which is correlated with pulse-width, period and age in such a way as to lead to the correlation we have noted.

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FIGURE CAPTIONS

Figure 1: Distribution of pulsars according to log $P_T^{2/3}$ and log $WP^{-2/3}$. S, C, D indicate the pulse type according to the Taylor, Manchester and Huguenin classification. The dotted line indicates the separation of the two clusters and the crosses are the center of the two clusters.

Figure 2: Histogram of likelihood ratios for 500 random data sets. The likelihood ratio of the pulsar data is 114.6. There are 72 data sets with likelihood ratio $\geq$ 114.
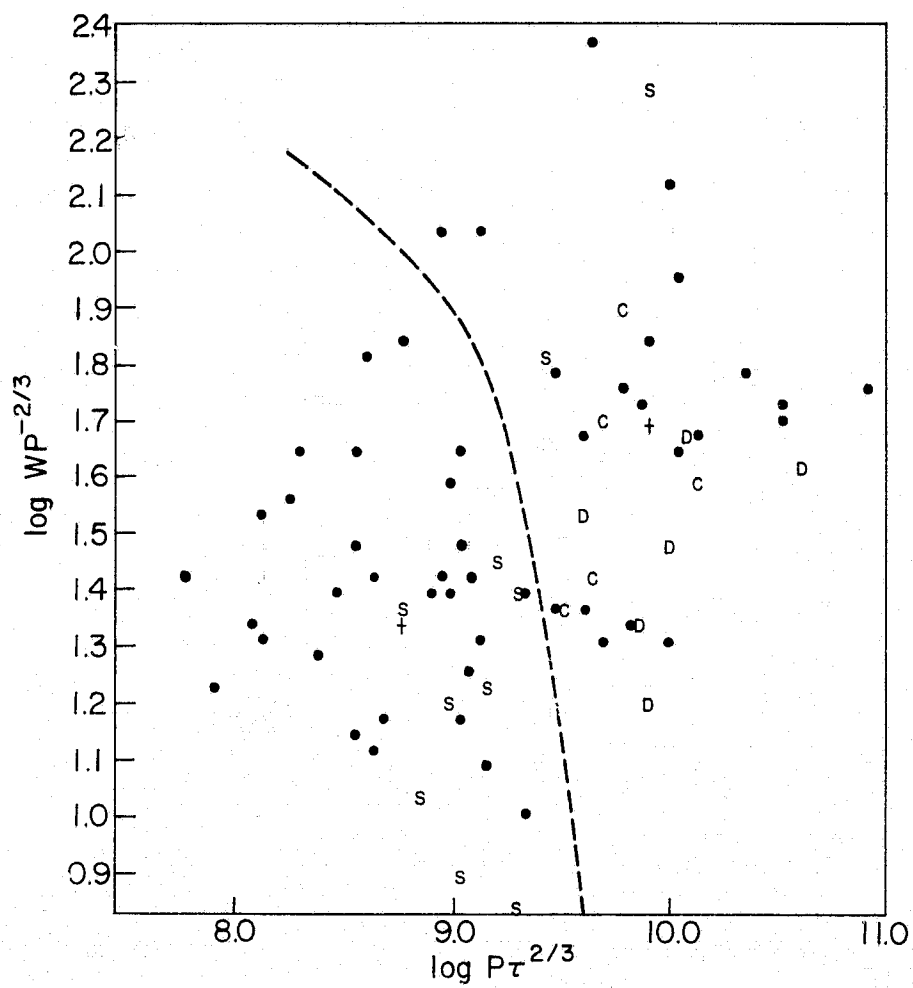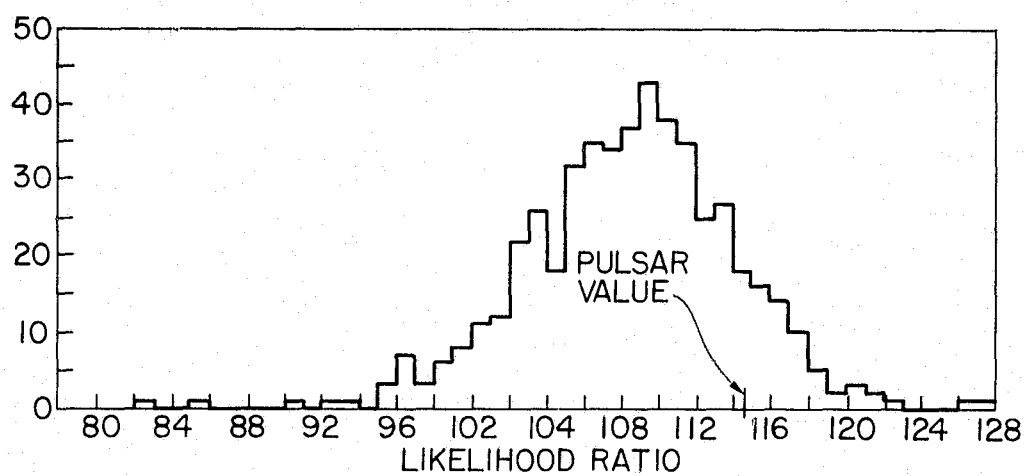
Figure 1

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Figure 2